"DESIGN OF SHEAR CONNECTORS IN COMPOSITE CONCRETE-STEEL BRIDGES"
DESIGN OF SHEAR CONNECTORS IN COMPOSITE CONCRETE-STEEL BRIDGES

Prepared for
MISSOURI STATE HIGHWAY DEPARTMENT

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in cooperation with
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ABSTRACT

The purpose of this investigation was to study an alternate design procedure for proportioning stud or channel shear connectors which would reduce the number of connectors required but would otherwise satisfy the AASHO specifications. It was proposed to design the shear connectors on the basis of the shear diagram resulting from the loading for maximum bending moment and use a factor of safety of 4.0 on the useful capacity of the connector. Six simple-composite beams for spans ranging from thirty to eighty feet were designed for H20-S16-44 loading according to the 1961 AASHO specifications except for the shear connectors. The beams were analyzed by a method, proven experimentally, which considered the slips at the interface of the steel and concrete slab by incorporating the load-slip behavior of studs as determined from pushout test data. The results showed that even though the designs had an average of thirty-three percent fewer connectors than designs using the 1961 AASHO specifications, they still conformed with these specifications. It was concluded that welded stud and channel shear connectors can be designed according to the shear diagram for the loading for maximum positive bending moment, using a factor of safety of 4.0 on the useful capacity of the connector, and still satisfy the 1961 and 1965 AASHO specifications.
ACKNOWLEDGEMENTS

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INTRODUCTION

The design of highway composite concrete-on-steel bridges has been performed according to the AASHO Standard Specifications for Highway Bridges (1)*. These specifications require that the number and spacing of the shear connectors be determined from an analysis based upon elementary beam theory coupled with empirical values of the resistance at working load of an individual shear connector. The spacing or pitch, \( p \), is given as

\[
p \leq \frac{Q}{S}
\]

where

\[
Q = \frac{Q_{uc}}{F.S.}
\]

\( Q_{uc} \) = The critical load capacity of one shear connector or one pitch of a spiral bar

\[
F.S. = \frac{1}{(1+C_v)} \{2.7(1+C_{mc}+C_{mi}C_S)-(C_{mc}+C_{mi})C_v\}
\]

\[
C_{mc} = \frac{\text{Max. Mom. due to D.L. acting on composite section}}{\text{Max. L.L. Mom.}}
\]

\[
C_{mi} = \frac{\text{Max. Mom. due to D.L. acting only on steel beam}}{\text{Max. L.L. Mom.}}
\]

\[
C_S = \frac{\text{Mom. of inertia of composite bm. at pt. of max. mom.}}{\text{Dist. from neutral axis to extreme tensile fiber}}
\]

\[
C_S = \frac{\text{Mom. of inertia of steel bm. at pt. of max. mom.}}{\text{Dist. from neutral axis to extreme tensile fiber}}
\]

* Numbers in parentheses refer to publications listed on page 6.
Vertical shear at section considered due to D.L. on comp. section

\[ C_V = \frac{\text{Vertical shear due to L.L.}}{} \]

\[ S = \frac{V_m}{I} \quad (d) \]

\[ V = \text{total shear due to L.L. & Impact & D.L. (which depends upon the use of temporary shoring, see section 1.9.5 of reference (1).} \]

\[ m = \text{The statical moment of the area above or below the interface about the centroidal axis on the transformed composite section.} \]

\[ I = \text{the moment of inertia of the transformed composite section about the centroidal axis.} \]

The specification stated that a factor of safety of 4.0 may be used in lieu of calculating it from the given formula (c). Because of the nature of the formula for the factor of safety (varies from point to point along the beam) many designers used a constant factor of safety of 4. This resulted in the use of a relatively large number of connectors which the designer felt was excessive. If the factor of safety were calculated it would almost always be less than 4.

The purpose of this paper is to investigate a design procedure which would reduce the number of connectors required and still satisfy the specifications.

**APPROACH**

The standard design procedure would essentially be to load the beam such that the maximum possible bending moment would occur and select a section, which would satisfy the strength and deflection requirements resulting from this loading. Then the envelope of maximum shear would
be determined from which the shear connectors would be designed.

Since the beam is primarily a flexural member and the function of the shear connectors is to transmit horizontal shear or the force in the slab from the internal couple or bending moment, and since highway design loadings are such that all connectors are not working at their useful capacity, it was decided to design the shear connectors on the basis of the shear diagram resulting from the loading for maximum bending moment and use a factor of safety of 4. The design was then checked for high shear loading.

Therefore, six composite beams were designed according to the AASHO specifications except for the shear connectors, which were proportioned from the loading for maximum moment, instead of from the loading for maximum shear. A typical design is given in the Appendix.

LIMITATIONS

The study was limited to simple spans loaded symmetrically. Only stud shear connectors with upset heads were used but the results are applicable to channels as well.

The beams were designed assuming there was no slip at the interface of the steel and concrete slab but in the analysis it was considered. The method of analysis has been reported elsewhere (2) and will not be restated. It was demonstrated (2) that this analysis accurately predicts the behavior of simple span composite beams. The solution involves an iterative technique accomplished on a digital computer and as the number of connectors increase the computer time increases. Because of this time factor, span lengths in excess of eighty feet were not analyzed.
PROCEDURE

The beams were designed for H20-S16-44 loading with a stringer spacing of 8'-8" and a 30 foot roadway. The design properties of the beams are shown in Table 1 and a typical design is completely described in the Appendix. As stated previously, the shear connectors were proportioned from the shear diagram of the loading for maximum bending moment using a factor of safety of 4. Obviously, the shear connectors at a particular point in the beam should be designed to resist the maximum possible shear that could occur at that point. The designs were, therefore, checked by loading the beams with the design loads determined from the shear envelope as developed in the Appendix. Since the computer program is thusfar applicable only to symmetrically loaded members, a pair of equal concentrated symmetrical loads were placed a distance of one-tenth of the span length from each support. The magnitude of the loads was equal to the live load plus impact design shear determined from the shear envelope at the one-tenth point of the span. The forces in the connectors were then determined from the analysis and combined with the dead load forces and compared with the allowable force obtained from formulas (b) and (c).

RESULTS

The results of the analysis at design loads compared extremely well with the values predicted in the design. Using beam no. 4 (60 ft. span) as a typical example, the analysis incorporating slip predicted a center line deflection due to L.L. + Imp of 0.552 inches compared with 0.548 inches using the elementary transformed section theory. The stress at
the bottom of the steel beam due to L.L. + Imp was 13,230 psi from the
analysis compared with 12,610 psi using the transformed section (about
5% difference).

The results of the analysis of the force carried by a connector
when subjected to its design shear load is shown in Table 2. It is
noted that although the connectors were designed from the loading for
maximum moment they are not overstressed when subjected to their
maximum possible design shear force (even with F.S. = 4). The reason
for this seemingly unreasonable result is that connectors closer to the
support (point of zero moment) are more highly stressed than those
located near the applied load, according to the analysis. Table 3 gives
the maximum force carried by a connector at any point in the beam for the
various loadings. In none of the connectors did the force exceed the
allowable as determined using the factor of safety from equation (c)
although as expected they did exceed the allowable with F.S. = 4 in
most cases.

Since this investigation was begun the AASHO specifications have
been revised such that equation (c) was eliminated and a uniform factor
of safety of 3.0 was adopted. It is interesting to compare the allowable
force per connector when determined using equation (c) with that specified
in the revised code (F.S. = 3). It is seen (Table 3) that in most cases
they are within 5% of each other.

A comparison of the theoretical number of shear connectors required
depending upon the design loading and connector factor of safety is
shown in Table 4. When using the loading for maximum moment, the
required number is considerably less (33% average) than the number obtained
by using the 1961 AASHO specifications and F.S. = 4. In beam no. 4, the
number of connectors required using the loading of the shear envelope and the factor of safety of equation (c) is 224. This number is about 17% more than the number based upon the moment loading but reasonably close to the number based upon the 1965 AASHO specifications (F.S. = 3).

CONCLUSIONS

From the results of this study the following conclusions are made:

(1) Stud and channel shear connectors can be designed according to the shear diagram for the loading for maximum positive bending moment, using a factor of safety of 4.0 on the useful capacity of the connector, and still satisfy the 1961 AASHO specification. This design will also satisfy the 1965 AASHO specification.

(2) The design calculations are easier and faster when the connectors are proportioned from the loading for moment and the number of connectors required are fewer than those required from the AASHO specifications. Therefore, this design method is more economical.

REFERENCES


<table>
<thead>
<tr>
<th>Beam No.</th>
<th>Span (Ft.)</th>
<th>Steel Beam Section</th>
<th>Slab Thickness (in.)</th>
<th>Effective Slab Width (in.)</th>
</tr>
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<tr>
<td>1</td>
<td>30</td>
<td>24WF76</td>
<td>7</td>
<td>84</td>
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<td>40</td>
<td>30WF99</td>
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</tr>
<tr>
<td>3</td>
<td>50</td>
<td>33WF130</td>
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<td>84</td>
</tr>
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<td>7.5</td>
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</tr>
<tr>
<td>5</td>
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<td>36WF300</td>
<td>7.5</td>
<td>90</td>
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<td>Beam No.</td>
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<td>Force per Conn. Due to Design Shear (lbs.)</td>
<td>Allow. Conn. Force (lbs.)</td>
<td>F.S. from Eq. (c)</td>
</tr>
<tr>
<td>----------</td>
<td>----------------------</td>
<td>------------------------------------------</td>
<td>--------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>2700</td>
<td>4020</td>
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<td>570</td>
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<td>6</td>
<td>8</td>
<td>2850</td>
<td>4870</td>
<td>3.29</td>
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</table>

1 The loading consists of two equal concentrated loads, symmetrically located a distance "a" from the support.

2 These values are the force in the connector located the distance "a" from the support and subjected to the maximum possible shear at this section (D.L. + L.L. + Imp.)

3 These values are the allowable force per connector using the factor of safety determined from equation (c).
<table>
<thead>
<tr>
<th>Beam No.</th>
<th>Stud Diam. (in.)</th>
<th>Load Position a (ft.)</th>
<th>Max. Force on Connector From Analysis (kips)</th>
<th>Allowable Force Per Connector (kips)</th>
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<tr>
<td></td>
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<td></td>
<td>F.S. Eq.(c)</td>
<td>F.S.=4</td>
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<td>3</td>
<td>3.81</td>
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<td>2.50</td>
<td>5.02</td>
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<tr>
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<td>7/8</td>
<td>7</td>
<td>4.65</td>
<td>5.10</td>
</tr>
<tr>
<td>6</td>
<td>7/8</td>
<td>8</td>
<td>4.06</td>
<td>5.15</td>
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</table>

TABLE 3 - COMPARISON OF MAXIMUM CONNECTOR FORCE WITH ALLOWABLE
### TABLE 4 - COMPARISON OF REQUIRED NUMBER OF CONNECTORS

<table>
<thead>
<tr>
<th>Beam No.</th>
<th>Span (Ft.)</th>
<th>Theoretical No. of Shear Connectors Based Upon Loading For Maximum</th>
<th>Stud Diam. (in.)</th>
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<tr>
<td></td>
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<td>Moment (F.S. =4)</td>
<td>Shear (F.S. =4)</td>
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<tr>
<td>1</td>
<td>30</td>
<td>126</td>
<td>210</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>168</td>
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<td>156</td>
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<tr>
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<td>60</td>
<td>192</td>
<td>280</td>
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<td>5</td>
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<td>306</td>
</tr>
<tr>
<td>6</td>
<td>80</td>
<td>264</td>
<td>366</td>
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</table>
APPENDIX

DESIGN OF TYPICAL INTERIOR STRINGER
1961 AASHO Standard Specifications for Highway Bridges

Span Length = 60'-0"  
Stringer Spacing = 8'-8"  
Roadway Width = 30'-0"

$ f' = 4,000 \text{ psi} $  
$ f_s = 20,000 \text{ psi} $  
n = 8

H20-S16-44 loading

Maximum Live Load Moment = $806.5'k$ (p.273 AASHO)

This moment is determined by standard truck loading and since the analysis can utilize only symmetrical loading, an equivalent lane loading will be determined.

Ratio of concentrated load for moment to uniform lane live load is

$$ \frac{18000}{640} = 28.1. $$

Therefore, the beam is loaded as shown with the requirement that

$$ M_{\text{max.}} = 806.5'k(\text{axle load}). $$

\[ M_{\text{max.}} = \frac{w(60)^2}{8} + \frac{28.1w(60)}{4} = 806.5 \]

$ w = 0.925k' \text{ for full lane width} $  

Distribution Factor for Bending = $ \frac{S}{5.5} = \frac{8.667}{5.5} = 1.576 \text{ per wheel} $ (sec.1.3.1)

So, the design live load moment = $ \frac{806.5}{2}(1.576) = 632'k $
Impact Factor, \( I = \frac{50}{60 + 125} = 0.27 \)

Assume slab thickness = 7 1/2"  Weight = 150 p.c.f.

D.L. moment of slab = \( \frac{7.5}{12} \cdot (8.67) \cdot 150 \cdot \frac{(60)^2}{8} = 366 \text{k} \)

Design moment = 632(1.37) + 366 = 1168 k

Try 36WF160  \( A = 47.09 \text{in}^2, d=36.00", I=9738.8 \text{in}^4 \)

\[ S = 541.0 \text{in}^3 \]

Moment = \( \frac{160(60)^2}{8} = 72 \text{k} \)

Effective Slab Width (Sec. 1.9.3)

1. \( \frac{L}{4} = \frac{60}{4} = 15" \)

2. Stringer spacing = 8'-8" = 104"

3. \( 12t = 12(7.5) = 90", \) use 90"

Transformed Section Properties

<table>
<thead>
<tr>
<th>Sec</th>
<th>A</th>
<th>Y</th>
<th>AY</th>
<th>AY^2 + Io</th>
</tr>
</thead>
<tbody>
<tr>
<td>slab</td>
<td>84.3</td>
<td>-3.75</td>
<td>-316</td>
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<tr>
<td>36WF160</td>
<td>47.1</td>
<td>+18.0</td>
<td>848</td>
<td>15260+9739</td>
</tr>
<tr>
<td>( \sum )</td>
<td>131.4</td>
<td></td>
<td>532</td>
<td>16448+10135</td>
</tr>
</tbody>
</table>

\[ \bar{y} = \frac{532}{131.4} = 4.05" \]

\[ I_x = 26,583 \text{in}^4 \]

\[ I_c = I_x - A(\bar{y})^2 = 26,438 - 131.4(4.05)^2 = 24,378 \text{in}^4 \]

Stresses

Steel D.L. WF section, \( f = \frac{M}{S} = \frac{72(12000)}{541} = 1600 \text{ psi} \)

L.L. + Imp. + Slab D.L., \( f = \frac{M_t}{I_c} = \frac{1168(12000)31.95}{24,378} = 18330 \text{ psi} \)

Max. \( f_s = 19,930 \text{ psi} < 20,000 \text{ psi OK} \)
Concrete

L.L. + Imp. + Slab D.L., \( f_c = \frac{Mc}{nI_c} = \frac{1168(12000)11.55}{8(24,378)} = 830 \text{ psi} \)

max. \( f_c = 830 \text{ psi} < 1600 \text{ psi} \) OK

Deflection due to L.L. + Imp.

Distribution factor = 1.288 (Missouri State Highway Specs., based upon the average distribution for moment and number of wheel lines divided by the number of stringers)

Uniform load, \( w = \frac{.935}{2} (1.288) 1.27 = .757k/\text{f.} \) per stringer

Concentrated load, \( P = \frac{28.1(.925)(1.288)1.27}{2} = 21.25k \)

\( \delta = \frac{5wL^4 + PL^3}{384EI_c} + \frac{PL^3}{48E I_c} \), \( E = 29 \times 10^6 \text{ psi}, I_c = 24,400 \text{in.}^4 \)

\( \delta = .548'' < .60'' = \frac{L}{1200} \) OK

Shear Connectors

Use units of 3- \( \frac{7}{8} '' \) studs, \( Q_{uc} = 16k \) per stud, F.S. = 4.0

\( Q = \frac{16(3)}{4} = 12k \) per unit

Spacing = \( \frac{Q}{S} = \frac{Q}{Vm/I_c} = p \)

\( m = (11.25)(7.5) \left( \frac{7.5}{2} + 4.05 \right) = 658 \text{in}^3 \)

\( p = \frac{12(24,400)}{V(658)} = 445 \frac{V}{\text{in.}} \)
To determine the spacing of the studs, the theoretical spacing is calculated at, say, 5 foot intervals along the span and a theoretical curve plotted to scale. From this curve the actual spacing is determined with the AASHO limitations of a 6" minimum and a 24" maximum spacing. The following spacing was adopted:
The design resulted in the use of 32 sets of 3-\(\frac{1}{8}\) studs per half of the beam or a total of 192 studs. Theoretically, the number of studs may be determined as follows:

The spacing, \(p = \frac{Q I}{V m}\)

and \(p V = \frac{Q I}{m} = \) constant

Now, \(p V\) is the area under the shear diagram taken by one set of studs and \(\sum(pV)\) is the total area under the shear diagram. If there are \(N\) sets of studs on the beam, then

\[
\sum(pV) = \sum\left(\frac{Q I}{m}\right) = N \left(\frac{Q I}{m}\right)
\]

and

\[
N = \frac{\sum(pV)}{(Q I)/m} = \frac{\text{Area under V-diagram}}{(Q I)/m}
\]

For the 60 ft. span,

\[
N = \frac{1}{445} \left\{ \frac{1}{2}(65.2 + 13.0)30(12) \right\} \times 2 = 63.3 \text{ sets}
\]
This results in the same number of studs as determined before. This approach will be used to determine the number of connectors required when using the shear envelope.

**Design of Shear Connectors From Shear Envelope**

The shear envelope is the curve such that the ordinate at any point represents the maximum possible shear at that point. Consider a point \( x \) in the beam shown,

\[
V_x = \frac{P}{L} x + \frac{1}{2} (x) \left( \frac{X}{L} (w) \right) = \frac{P}{L} x + \frac{W_x}{2L} \tag{2}
\]

The dead load shear at B is
The total shear at B due to live load plus impact plus dead load is

\[ V_x = w_D \left( \frac{L}{2} \right) - w_D (L-x) = w_D \left( x - \frac{L}{2} \right) \]  

Equation (4) is the equation of the shear envelope for a simple span highway stringer using an equivalent lane load and moving concentrated load to represent the standard truck loading.

For the 60 ft. beam under consideration, the adjusted concentrated load for live load shear is

\[ \frac{26000}{640} \times \frac{640}{925} = 37,600 \text{ lbs.} \]

Using the same distribution factor as for bending, the load per stringer is

\[ P = 37,600 \left( \frac{1}{2} \right) 1.576 = 32,000 \text{ lbs.} \]

The uniform live load per stringer is

\[ w_L = 925 \left( \frac{1}{2} \right) 1.576 = 728 \text{ lbs. per ft.} \]

From equation (4) with \( w_D = 813 \text{ lbs. per ft.} \), the following shears are calculated in kips:
<table>
<thead>
<tr>
<th>x</th>
<th>60'</th>
<th>54'</th>
<th>48'</th>
<th>42'</th>
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<td>9.77</td>
<td>4.88</td>
<td>0</td>
</tr>
<tr>
<td>$V_{LL}$</td>
<td>51.40</td>
<td>44.20</td>
<td>37.80</td>
<td>31.50</td>
<td>25.65</td>
<td>20.30</td>
</tr>
<tr>
<td>$V_{LL+I}$</td>
<td>65.35</td>
<td>56.20</td>
<td>48.00</td>
<td>40.00</td>
<td>32.60</td>
<td>25.80</td>
</tr>
<tr>
<td>$V_{DL+LL+I}$</td>
<td>89.75</td>
<td>75.72</td>
<td>62.65</td>
<td>49.77</td>
<td>37.48</td>
<td>25.80 kips</td>
</tr>
</tbody>
</table>

A plot of the above shears results in a curve which is very closely approximated by a straight line. Using $4-\frac{7}{8}$" studs per set and a factor of safety of 4.0, equation (b) becomes

$$Q = \frac{16}{4} = 4 \text{ kips per stud or } 16 \text{ kips per set and equation (a) is}$$

$$p \leq \frac{Q}{s} = \frac{16}{(VQ)/l} = \frac{16(24,400)}{658V} = \frac{593}{V}$$

Therefore,

$$N = \frac{\text{Area under V-Diagram}}{593}$$

$$N = \frac{1}{593} \left\{ \frac{1}{2} (25.80+89.75) 30(12) \right\} x 2 = 70.0 \text{ sets}$$

or a total of 280 studs are theoretically required.

**Allowable Factor of Safety**

The allowable factor of safety is given by equation (c) as follows:

$$F.S. = \frac{1}{1+C_v} \left\{ 2.7(1+C_{m_c}+C_{m_i} C_s) - (C_{m_c}+C_{m_i}) + C_v \right\}$$
For the 60 ft. beam under consideration, the following constants are determined:

\[ C_{mc} = \frac{366}{632} = 0.578 \]

\[ C_{mi} = \frac{72}{632} = 0.114 \]

\[ C_S = \frac{24,400/32}{9739/18} = 1.41 \]

\[ C_V = \frac{V_{DL}}{V_{LL}} \] and varies from point to point along the span.

The value of \( C_V \) and the factor of safety are determined at six-foot intervals as shown below:

<table>
<thead>
<tr>
<th>x</th>
<th>60</th>
<th>54</th>
<th>48</th>
<th>42</th>
<th>36</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_V )</td>
<td>0.474</td>
<td>0.442</td>
<td>0.388</td>
<td>0.310</td>
<td>0.190</td>
<td>0</td>
</tr>
<tr>
<td>F.S.</td>
<td>3.04</td>
<td>3.08</td>
<td>3.17</td>
<td>3.30</td>
<td>3.53</td>
<td>4.0</td>
</tr>
</tbody>
</table>